# COMPARISON OF PRACTICAL APPROACHES FOR MODELLING SHEARWALLS IN STRUCTURAL ANALYSES OF BUILDINGS

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## ABSTRACT:

Modeling shearwalls is very important issue for static and dynamic analyses of building structures. For the purpose of finite elements modelling, different techniques utilizing either shell elements or combination of frame elements can be used. Shell elements formulations generally consist of out-of-plane (plate) and in-plane (membrane) degrees of freedom. The membrane element with drilling degrees of freedom was a challenge for the engineering community for many decades. The membrane elements generally combined with plate elements to form a “shell element” that has six degrees of freedom at each node and an in-plane rotational degree of freedom, which makes it compatible with three-dimensional beam-type finite element. This approach was successful and many analysis software have adopted various formulations for the shell elements. In practical engineering, although the shell element appears to have full compatibility with three-dimensional beam element, some limitations in the formulation were identified. Although drilling rotations allow introducing external loads in the form of drilling moments, analytical results show inconsistency and sensitivity to mesh sizes and loading conditions. In this study, different approaches of modeling the shearwalls in structural analyses of buildings are discussed and compared. The effect of mesh size of shell elements on the bending moment of attached beams will be emphasized and different practical solution will be investigated.

**KEYWORDS:** Shearwalls, shell element, membrane element, finite elements, drilling degree of freedom

## 1. INTRODUCTION

In the last decades, the finite elements (FE) method is used as an effective tool for static and dynamic analysis of structures. For the regions of high seismicity, the use of shearwalls in building structures to resist the lateral forces is very common in engineering practice. The shearwalls within the building structures are generally modeled by either a composition of frame elements or a mesh of shell elements. Modeling shearwalls with frame elements are used very extensively in building analysis due to its simplicity and the capability to use linear and nonlinear features of the existing design software. Utilizing shell elements for shearwalls was greatly enhanced after the extensive researches done in the last three decades for stable and compatible shell formulations with the three-dimensional finite element models. In general, shell elements with six degrees of freedom per node are ideal for the analysis of planar structural elements. Shell elements are divided into two elements in terms of the degrees of freedoms. The two in-plane rotational and perpendicular displacement degrees of freedom form the plate element and the in-plane displacements and drilling rotation give rise to the membrane element. Membrane elements with drilling degrees of freedom are of great practical interest, since they provide a possibility for constructing a fully compatible model of a complex structural system in which they appear combined with beam elements. The classical FE formulation to develop membrane elements with drilling degrees of freedom was unsuccessful. Many efforts to develop membrane elements with drilling degrees of freedom were made during the period 1964–1975, which came out with inconclusive results. It was Allman (1988), who introduced the concept of “the quadrilateral finite element with vertex rotations”. Later, Hughes and Brezzi (1989) presented a variational formulation employing an independent rotation field.

Ibrahimbegovic (1990), (1995) presented membrane elements with drilling degrees of freedom based on a variational formulation which employs an independent rotation field. These formulations were resulted in the development of two new membrane elements, namely MQ2 and MQ3 and the elements exhibited good performance over a set of problems. Since the addition of drilling rotations to membrane elements has been a difficult task, involving complex formulation, its effect in the analytical results must be assessed. Although drilling rotations allow inputting external loads in the form of drilling moments, analytical results show inconsistency and sensitivity to mesh sizes and loading conditions. Generally, the membrane element with translational degrees of freedom has low in accuracy and that created the challenge for the engineering community to include the rotational degrees of freedom at every node. The investigations of shell elements formulations still attract many recent researchers. A comprehensive literature can be found in Yang et al (2000) and Paknahad (2007).

In this study, different practical approaches utilizing frame and shell elements for modeling the shearwalls in structural analyses of buildings are explained and compared. The effects of frame elements rigidity and the mesh size of shell elements on the design forces of the shearwall for both gravity and lateral loads are studied.

## 2. MODELLING OF SHEAR-WALLS IN BUILDING ANALYSES

Application of the finite element method for the analysis of building structures with shearwalls requires an understanding of the approximations involved in the modeling assumptions to build these elements. The two modeling procedure and assumptions are explained below

### *2.1. Shell Elements Based Model*

The shell element can be used efficiently for the analysis of building structures with shearwalls. The shell element considered in most of the design software has six degrees of freedom at each node and an in-plane rotational degree of freedom, which makes it compatible with three-dimensional beam-type finite element models. It is worth to know that a bilinear shape functions are used to define the displacement field of the quadrilateral elements, Wilson (2002). Therefore, shearwall modeling requires a mesh discretization in order to get realistic behavior. The advantage of using shell elements is the ability to model very long, interacting and complex shearwalls within the three dimensional model. The optimal size of the mesh and the effects of mesh size on the results are shown in the numerical examples below. Although the shell element formulations include the drilling degree of freedom, analytical results show inconsistency and sensitivity of the drilling moment to mesh sizes and loading conditions. This shortcoming has significant effects on the bending moment of the in-plane beams connected to the shearwall. To resolve this problem, in engineering practice, the beam connecting to shear wall are generally modeled to some extend inside the shearwall shell elements.

### *2.2. Frame Elements Based Model*

The shearwalls are modeled using a set of frame elements. The most common modeling technique is to use a composition of mid-pier frame to represent the shearwall stiffness and a horizontal frame (rigid arm) to allow proper connections with intersecting beams and slab components. The most critical point for this model is the proper selection of rigidity and stiffness property for the horizontal frame. Infinite rigidity of the upper frame can highly overestimate the bending moments especially at the connecting beams. This model is used widely in practice to model planar shearwalls in building structures for linear and nonlinear analyses. This model might have no reliable results for very long, interacting or complex shearwalls with openings.

## 3. NUMERICAL EXAMPLE

The typical structural plan of the example building is shown in Figure 1 and column, wall and beam cross-section dimensions and the slab thickness are given in Table 1. The cross-sections of the vertical structural members are assumed constant over the entire height. The building has five storeys with a typical storey height of 3.2 m. Three dimensional physical and analytical models are shown in Figure 2. Reinforced Concrete of grade C20 is used for structural members. Modulus of elasticity value is used as 2.85x10$^7$ kN/m$^2$ according to Turkish Reinforced Concrete Design Code (TS500). Uncracked section properties are used throughout the analyses. The storey masses are formed using appropriate distribution of the slabs loads. The slab loads composed of self weight (G) and 30% of the live load (Q), where, G = own weight + 1 kN/m$^2$, Q = 2 kN/m$^2$. The centre of mass of the building is calculated based on the mass distribution at each node. Orion (2008), structural design software is utilized for three dimensional modeling and analyses of the example building. The shell element formulations in the software are based on formulations proposed by Wilson (2002). Design seismic load is calculated using the Equivalent Static Load Method based on the Turkish Earthquake Code, (2007). The building is located in the first seismic zone, with local soil profile Z2 and building importance coefficient (I) equal to 1. As it is suggested by the code, the structural behavior factor (R) is considered to be 7. This factor reduces to 6.11 after wall-frame interaction checks according to code. The design spectrum curve is given in Figure 1. The accidental eccentricity is ignored in the seismic loading to directly observe the lateral load effect on the walls.

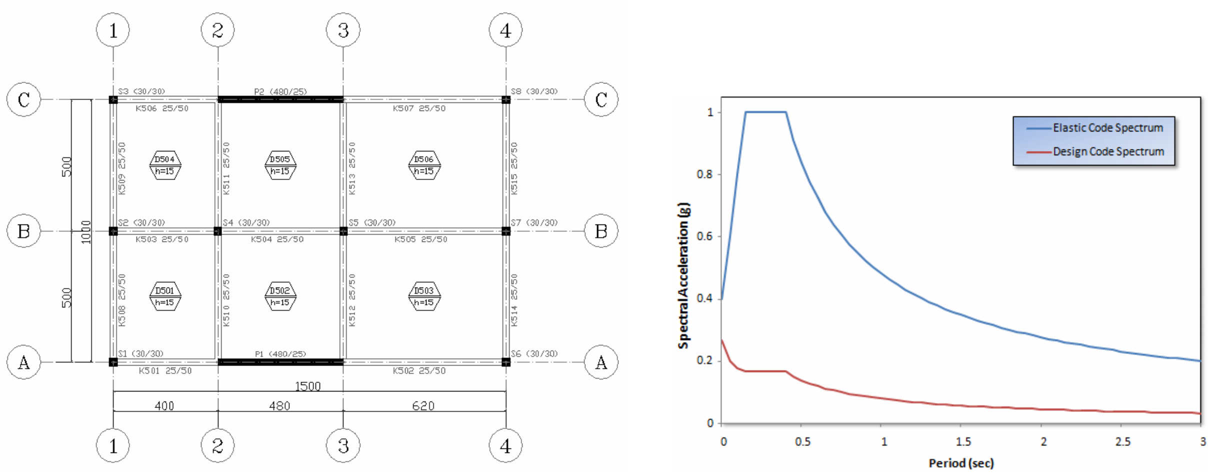

Figure 1: Structural plan of the example building and Design response spectrum used in the analysis

Table 1 Members dimensions of the example building

| | |
|---|---|
| ***Columns (mm)*** | 300x300 |
| ***Walls (mm)*** | 4800x250 |
| ***Beams (mm)*** | 250x500 |
| ***Slab Thickness (mm)*** | 150 |

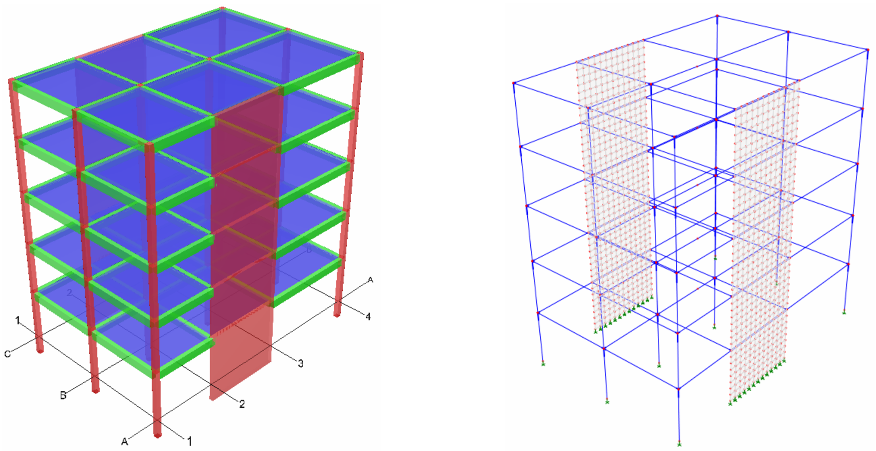

Figure 2: Three dimensional physical and analytical model of the example building

### 3.1. *Explanation of the Models and Analysis Sets*

Three analysis sets for the example building are considered using different modelling techniques of the shearwalls: 1) Shearwalls are modelled with shell elements with varying mesh sizes from coarse to fine. 2) Rigid beams along the wall top chords are introduced to "shell elements" model to enhance drilling moment computations. 3) Shearwalls are modelled by frame element as (MidPier Model) and upper frame element "rigid arm" is introduced to ensure connectivity with connecting beams and slab members. For the sake of comparison, various sectional properties of these rigid arms are considered. For each set of the above models, shearwall internal forces are computed for gravity (1.4G + 1.6Q) and earthquake loading in X direction (EX). In addition, the internal forces of the beams connecting to shearwalls in both in-plane and out-of-plane orientations are tabulated.

### 3.2. *Modelling of Shearwalls Using Shell Elements*

#### 3.2.1. *Shell Elements Only*

In this set, 4.8 m wide shearwalls are modelled with shell elements with varying mesh sizes from 160x160 cm to 20x20 cm. Results are given in Tables 2-3. The analyses results shows dramatic change (≈10 times) in M3 moment with the change of mesh size of shell elements for both shearwall and beams along major direction under gravity loads.

Table 2: Wall Internal Forces (kN,m) for "Shell Elements Only" Model

| | **Gravity Loading (1.4G+1.6Q)** | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| Shell 160x160 | -2010.85 | -10.72 | -17.35 | -19.46 | 391.23 |
| Shell 80x80 | -2016.92 | -11.76 | -14.84 | -16.62 | 114.51 |
| Shell 40x40 | -1943.29 | -12.91 | -13.12 | -14.70 | 80.89 |
| Shell 20x20 | -1903.66 | -13.63 | -11.77 | -13.18 | 36.32 |

| | **Earthquake Loading (EX)** | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| Shell 160x160 | -4.52 | 54.53 | 0.00 | 0.00 | 533.58 |
| Shell 80x80 | -3.33 | 54.56 | 0.00 | 0.00 | 570.32 |
| Shell 40x40 | -2.77 | 54.58 | 0.00 | 0.00 | 578.42 |
| Shell 20x20 | -2.45 | 54.60 | 0.00 | 0.00 | 585.77 |

Table 3: Beam Internal Forces (kN, m) for "Shell Elements Only" Model

| | **Beam along major wall direction** | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| Shell 160x160 | 0.96 | -61.08 | -67.48 | -0.02 | 0.56 | 1.85 |
| Shell 80x80 | 1.27 | -58.17 | -53.94 | -0.03 | 0.35 | 0.83 |
| Shell 40x40 | 1.51 | -52.22 | -26.91 | -0.03 | 0.28 | 0.55 |
| Shell 20x20 | 1.69 | -48.80 | -11.38 | -0.03 | 0.23 | 0.30 |

| | **Beam framed in minor wall direction** | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| Shell 160x160 | 1.85 | -61.31 | -44.63 | 0.06 | 0.25 | 0.51 |
| Shell 80x80 | 0.53 | -59.57 | -38.83 | 0.05 | 0.25 | 0.49 |
| Shell 40x40 | -2.04 | -58.36 | -34.75 | 0.08 | 0.24 | 0.45 |
| Shell 20x20 | -3.52 | -57.39 | -31.52 | 0.10 | 0.23 | 0.42 |

### 3.2.2. *Shell Elements With Rigid Beams Along Wall Top Chord*

Rigid beams are defined along the wall top chords. The torsional constant and moment of inertias (J, I2, I3) are the only parameters used to define the rigid beam cross section. These properties of rigid beams are considered to be the same as the beams framing into the shearwall. The purpose of these rigid beams is to minimize the mesh sensitivity effect caused by the inadequacy of drilling degree of freedom formulations. Illustration of the model is shown in Figure 3 and the results are given in Tables 4-5. As it can be seen clearly in the results, stable estimation of M3 moment and other internal forces for both shearwalls and connecting moment is achieved. Mesh size of 160x160 generally give %5-%10 difference from the finer meshes.

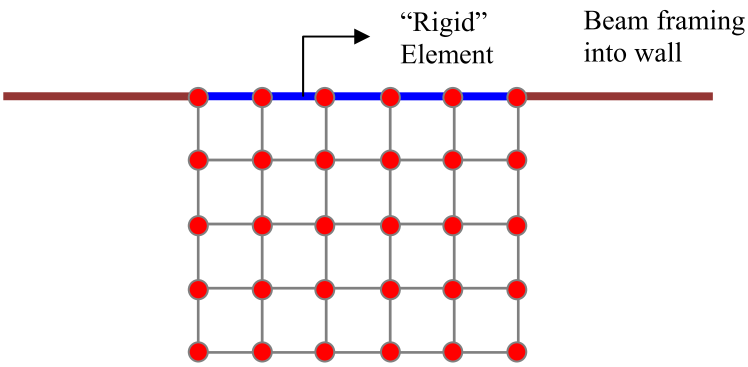

Figure 3: Rigid element defined along the top chord of the wall

Table 4: Wall Internal Forces (kN, m) for Shell Elements With Rigid Beams Along Wall Top Chord

| | Gravity Loading (1.4G+1.6Q) | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| Shell 160x160 | -2088.75 | -10.60 | -18.96 | -21.04 | 338.82 |
| Shell 80x80 | -2078.53 | -10.77 | -17.65 | -19.50 | 317.52 |
| Shell 40x40 | -2068.79 | -10.82 | -17.07 | -18.84 | 316.87 |
| Shell 20x20 | -2063.86 | -10.86 | -16.86 | -18.59 | 315.18 |

| | Earthquake Loading (EX) | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| Shell 160x160 | -5.26 | 54.53 | 0.00 | 0.00 | 530.66 |
| Shell 80x80 | -5.17 | 54.49 | 0.00 | 0.00 | 533.10 |
| Shell 40x40 | -5.09 | 54.48 | 0.00 | 0.00 | 533.89 |
| Shell 20x20 | -5.03 | 54.48 | 0.00 | 0.00 | 534.66 |

Table 5: Beam Internal Forces (kN, m) Elements With Rigid Beams Along Wall Top Chord

| | Beam along major wall direction | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| Shell 160x160 | 0.74 | -63.35 | -77.58 | -0.02 | 0.55 | 1.78 |
| Shell 80x80 | 0.88 | -62.72 | -74.75 | -0.02 | 0.53 | 1.72 |
| Shell 40x40 | 0.94 | -62.30 | -72.94 | -0.02 | 0.53 | 1.72 |
| Shell 20x20 | 0.96 | -62.02 | -71.69 | -0.02 | 0.53 | 1.70 |

| | Beam framed in minor wall direction | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| Shell 160x160 | 2.80 | -62.47 | 48.51 | -0.05 | 0.26 | 0.55 |
| Shell 80x80 | 2.55 | -61.47 | -45.35 | -0.04 | 0.25 | 0.53 |
| Shell 40x40 | 2.40 | -61.00 | -43.90 | -0.04 | 0.25 | 0.52 |
| Shell 20x20 | 2.29 | -60.83 | -43.37 | -0.04 | 0.25 | 0.52 |

### 3.2.3. *Shell Elements With Rigid Beams Penetrating Along One Mesh*

Rigid beams are defined to penetrate into the wall for a length of one single mesh. Sectional properties and purpose of use are same with previous section. Illustration of the model is shown in Figure 4. Results are given in Tables 6-7. As it can be noticed from the results, using one penetrating rigid element along the top mesh give good results for coarse meshes. For finer mesh (20x20) 15% differences are exist in M3 moments for shearwalls and beams along major direction.

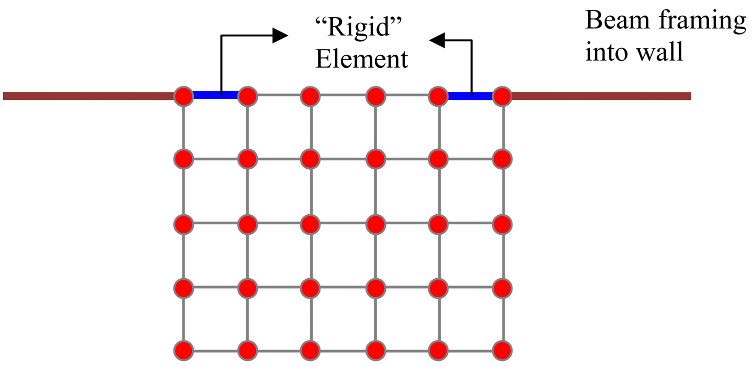

Figure 4: Rigid element penetrating along one mesh length

Table 6: Wall Internal Forces (kN, m) for Shell Elements for Rigid Beams with Penetrating Along One Mesh

| | **Gravity Loading (1.4G+1.6Q)** | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| Shell 160x160 | -2084.92 | -10.61 | -18.96 | -21.04 | 342.25 |
| Shell 80x80 | -2077.83 | -10.8 | -17.62 | -19.48 | 313.39 |
| Shell 40x40 | -2060.99 | -10.85 | -16.72 | -18.51 | 314.06 |
| Shell 20x20 | -2020.12 | -11.34 | -15.72 | -17.47 | 272.22 |

| | **Earthquake Loading (EX)** | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| Shell 160x160 | -5.19 | 54.53 | 0.00 | 0.00 | 530.89 |
| Shell 80x80 | -5.13 | 54.49 | 0.00 | 0.00 | 533.86 |
| Shell 40x40 | -5.03 | 54.48 | 0.00 | 0.00 | 534.96 |
| Shell 20x20 | -4.29 | 54.50 | 0.00 | 0.00 | 545.83 |

Table 7: Beam Internal Forces (kN, m) for Shell Elements with Rigid Beams Penetrating Along One Mesh

| | **Beam along major wall direction** | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| Shell 160x160 | 0.74 | -63.21 | -76.97 | -0.02 | 0.55 | 1.78 |
| Shell 80x80 | 0.89 | -62.63 | -74.36 | -0.02 | 0.53 | 1.70 |
| Shell 40x40 | 0.98 | -62.07 | -71.90 | -0.02 | 0.53 | 1.71 |
| Shell 20x20 | 1.09 | -59.22 | -59.05 | -0.02 | 0.49 | 1.48 |

| | **Beam framed in minor wall direction** | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| Shell 160x160 | 2.74 | -62.47 | -48.52 | -0.05 | 0.26 | 0.55 |
| Shell 80x80 | 2.51 | -61.46 | -45.29 | -0.04 | 0.26 | 0.54 |
| Shell 40x40 | 2.31 | -60.72 | -43.02 | -0.04 | 0.25 | 0.51 |
| Shell 20x20 | 1.11 | -59.94 | -40.58 | -0.02 | 0.24 | 0.49 |

### 3.3. *Modelling of Shearwalls Using Frame Elements*

In this model set, different section properties are used for the rigid arms element. Thickness of the rectangular rigid arm section can be considered the same as the wall itself. Different models are considered utilizing various rigid arm depth : half a storey height, a whole storey height, two times a whole storey height and ten times a whole storey height. Illustration is given in Figure 5 and results are given in Tables 8-9. The results show significant changes for M3 moment of the beam frame connected to the minor direction of the shearwall. Rigid arm with one height story depth give the most consistent results in comparison with "shell elements" models.

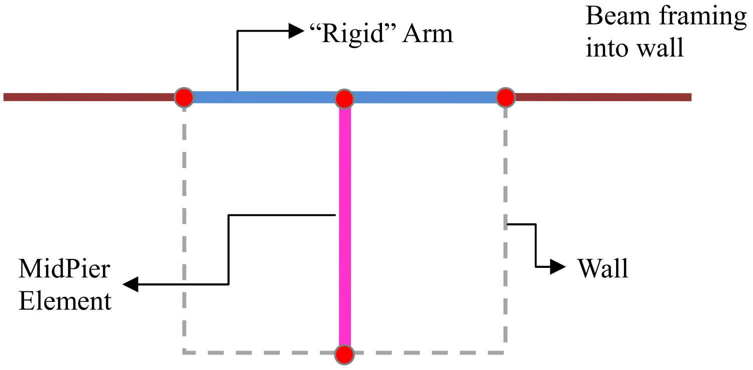

Figure 5: MidPier Model

Table 8: Wall Internal Forces (kN, m) for Modelling of Shearwalls Using Frame Elements

| | Gravity Loading (1.4G+1.6Q) | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| MidPier_Half | -2000.99 | 10.29 | -10.20 | 10.78 | 365.62 |
| MidPier_1St | -2086.37 | 10.28 | -15.66 | 16.56 | 347.66 |
| MidPier_2St | -2128.52 | 10.35 | -19.97 | 21.11 | 337.96 |
| MidPier_10St | -2172.20 | 10.43 | -25.07 | 26.51 | 328.52 |

| | Earthquake Loading (EX) | | | | |
|---|---|---|---|---|---|
| **Model** | **N** | **V2** | **V3** | **M2** | **M3** |
| MidPier_Half | -5.17 | -54.47 | 0.00 | 0.00 | 532.77 |
| MidPier_1St | -5.76 | -54.45 | 0.00 | 0.00 | 524.86 |
| MidPier_2St | -5.82 | -54.45 | 0.00 | 0.00 | 522.62 |
| MidPier_10St | -5.78 | -54.45 | 0.00 | 0.00 | 520.79 |

Table 9: Beam Internal Forces (kN, m) for Modeling of Shearwalls Using Frame Elements

| | Beam along major wall direction | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| MidPier_Half | 1.48 | -62.08 | -73.09 | -0.02 | 0.53 | 1.72 |
| MidPier_1St | 1.07 | -64.77 | -84.25 | -0.02 | 0.56 | 1.82 |
| MidPier_2St | 0.58 | -65.18 | -85.93 | -0.01 | 0.56 | 1.83 |
| MidPier_10St | -0.04 | -65.28 | -86.30 | 0.00 | 0.56 | 1.82 |

| | Beam framed in minor wall direction | | | | | |
|---|---|---|---|---|---|---|
| | **Gravity (1.4G + 1.6Q)** | | | **Earthquake (EX)** | | |
| **Model** | **T** | **V2** | **M3** | **T** | **V2** | **M3** |
| MidPier_Half | 2.79 | -55.12 | -26.72 | -0.04 | 0.19 | 0.34 |
| MidPier_1St | 3.52 | -59.91 | -40.55 | -0.05 | 0.25 | 0.52 |
| MidPier_2St | 3.60 | -63.22 | -51.11 | -0.05 | 0.29 | 0.66 |
| MidPier_10St | 3.60 | -67.04 | -63.54 | -0.05 | 0.35 | 0.86 |

## 4. DISCUSSIONS AND CONCLUSIONS

Based on numerical results for different buildings models and shearwalls configurations and the different analyses set results of the example building, the following conclusions can be made

- In modeling shearwalls with “shell elements”, the drilling moment of the shearwalls and the bending moment of the in-plane connected beams are changed dramatically with mesh density. For finer meshes 10 times reduction of the drilling moment can be estimated.
- Introduction of top chord frame stabilise the results considerably. Good estimation of the properties of the top chord frame is very important not to affect the overall stiffness of the structural system. Best results are obtained using a top chord frame element to enhance the fixity of the beams framing into the shearwall. Using one penetrating rigid element along the top mesh give good results for coarse meshes. For finer mesh (20x20) 15% differences are exist in drilling moments for shearwalls and bending moments of the beams along major direction.
- The size of the “shell elements” has considerable effects. Mesh size of 160 cm generally give %5-%10 difference from the finer meshes. The ideal mesh size for the example building was in the range of 50cm-100cm in terms of numerical accuracy and computational efficiency.
- In modeling shearwalls with frame elements (“midpier” model), the different properties for the rigid arms generate 5 to 15 percent differences in the internal forces of the shearwalls and beams framed in major direction. Significant variations are observed (more than two times) in the bending moment of the beams framed in minor direction .
- Rigid arm with one height story depth give the most reasonable results in comparison with “shell elements” models.
- It should be noted that the torsional stiffness of the top chord used for shell models should be set to a negligible level not to affect the fixity provided to the beams framed into the wall along minor direction.

## REFERENCES

Allman, D. (1988). A quadrilateral finite element including vertex rotations for plane elasticity problems. *Int. J. Num. Met. Engr* **26, 717-739**.

Henry T. Y. Yang, S. Saigal, A. Masud and R. K. Kapania (2000) "A survey of recent shell "nite elements", Int. J. Numer. Meth. Engng. 47, 101-127

Hughes, T.J.R., Brezzi, F. (1989). On drilling degrees of freedom. Comp. Methods Appl. Mech. Engng. 72: 105-121.

Ibrahimbegovic, A. and Frey, F. (1995). Variational principles and membrane finite elements with drilling rotations for geometrically non-linear elasticity. Int. J. Num. Met. Engr 38: 1885-1900.

Ibrahimbegovic, A., Taylor, R., and Wilson, L.(1990). A robust quadrilateral membrane finite element with drilling degrees of freedom. Int. J. Num. Met. Engr 30: 445-457.

Orion Building Design Software (2008), (Version 15) Software for Finite Element Analysis, Design and Detailing for Reinforce Concrete Buildings, CSC (UK) Ltd., Pudsey, UK / Prota Software Ltd., Ankara, Turkey.

Paknahad, M.A., Noorzaeia, J., Jaafara, M.S. and A. Thanoon, W.A. (2007). Analysis of shear wall structure using optimal membrane triangle element. Finite Elements in Analysis and Design, 43:11-12, 861-869

Turkish Earthquake Code for Buildings in Hazardous Areas, DBYBHY, (2006), Ministry of Public Works and Settlement Ankara.

Wilson, E.L. (2002). Three-Dimensional Static and Dynamic Analysis of Structures, A Physical Approach With Emphasis on Earthquake Engineering, Third edition 2002, Computers and Structures Inc, U.S.A.